The Taylor Series Expansion and An Introduction to Mathematica

à Purpose:

Our first excursion into Mathematica will be simple, for more detail on Mathematica I suggest Wolfram, S. *Mathematica: A System for Doing Mathematics on Computer* or Blachman, N. *Mathematica: A Practical Approach*. *The purpose of this notebook is to show the remainder term of a second order Taylor series expansion of a quadratic funciton.*

à The Mathematics

The first step is to declare a third order function. In this function x1 is the variable and a,b, and c are parameters.

```
In[1]:= g1 = x1^3 + a x1^2 + b x1 + c
```

```
Out[1]= c + b x1 + a x1^2 + x1^3
```

Given the original function, the second order Taylor expansion for g(x1) at x0 is:

```
In[2]:= fg1 = Ix0^3 + a x0^2 + b x0 + cM + I3 x0^2 + 2 a x0 + bM Hx1 - x0L + 1/2 H6 x0 + 2 aL Hx1 - x0L^2
```

```
Out[2]= c + b x0 + a x0^2 + x0^3 + Hb + 2 a x0 + 3 x0^2L H-x0 + x1L + 1/2 H2 a + 6 x0L H-x0 + x1L^2
```

A useful exercise to learn Mathematica is to automate the derivation from the first equation, but this will be left to the student. Note that the first term on the right hand side of the equation is simply g(x0). Instructing Mathematica to simplify this expression yields:

```
In[3]:= Simplify@fg1D
```

```
Out[3]= c + x0^3 - 3 x0^2 x1 + 3 x0 x1^2 + x1 Hb + a x1L
```

Subtracting this result from the orginal expression g(x1) yields the error of approximation:

```
In[4]:= g1 - fg1
```

```
Out[4]= -b x0 - a x0^2 - x0^3 + b x1 + a x1^2 + x1^3 - Hb + 2 a x0 + 3 x0^2L H-x0 + x1L - 1/2 H2 a + 6 x0L H-x0 + x1L^2
```

```
In[5]:= Simplify@%D
```

```
Out[5]= -Hx0 - x1L^3
```

Note that if x1-x0<1 then the error term converges to zero faster than x1 approaches x0.