Alternative Basis and Spanning Spaces

à Derivation of an alternative set of spanning vectors

As demonstrated in the MatLab program class1.m, the vector (14,39,54)' can be written as a linear combination of vectors (1,9,12)' and (4,7,10)'. However, the two spanning vectors are somewhat unappealing. For example, a graph of the space spanned by the two vectors is somewhat difficult to draw. Hence, we are interested in deriving two vectors which span the same space with somewhat more desirable characteristics. As a first step in this process, let us assume that we want to derive a vector (x,y,z)' as a linear combination of (1,9,12)' and (4,7,10)'. Specifically, for a given (x,y,z)' we want to derive that a and b such that a(1,9,12)'+b(4,7,10)'=(x,y,z)'. First note that a+4b=x. Solving for this relationship in terms of a we get:

```
step1 = Solve@a + 4 b == x, aD
```

```
{{a -> -4 b + x}}
```

This is what is known as a mathematica rule. We can use this rule in the second step which is to solve for b using the information in 9a+7b=y, or

```
step2 = Solve@9 a + 7 b == y •. step1, bD
```

```
        9 x - y
{{b -> -------}}
          29
```

The "/." denotes "such that" or "substitute". It instructs Mathematica to substitute the rule from step 1 into the expression. Next, the rule implied in step 2 can be substitued back into step 1 yielding step 3:

```
step3 = Simplify@step1 •. step2D
```

```
         -7 x + 4 y
{{{a -> ----------}}}
             29
```

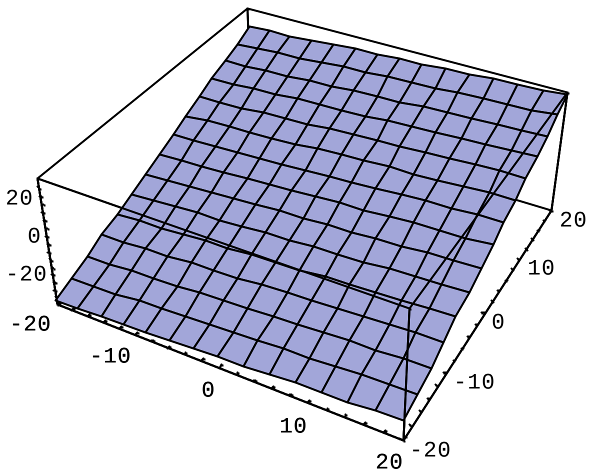
The value for z implied by these two equation is then derived by substituting step 2 and step 3 into 12a+10b=z

```
step4 = Simplify@12 a + 10 b == z •. step2 •. step3D
```

```
   2 (3 x + 19 y)
{{{-------------- == z}}}
         29
```

Thus, an alternative depiction of the space spanned by the vectors (1,9,12)' and (4,7,10)' is given by the spanning vectors (1,0,-6/29)' and (0,1,38/29)'. One advantage of this formulation is that it allows for the graphical depiction of the spanning space

Plot3DA$\frac{1}{29}$ H6 x + 38 yL, 8x, -20, 20<, 8y, -20, 20<E;



à Confirming the equivalence of the Spanning Space

A quick way to confirm the equivalence of the spanning space is to show that every point spanned by the original vectors is also a linear combination of the two new vectors. Instead of showing this for every point, we will show it for the point (14,39,54)' which was shown in class1.m to be linearly described by the two original vectors. To do this, we take x=14 and y=39. The corresponding z is then

$\frac{14\ 6}{29} + \frac{39\ 38}{29}$

54

Working our way backward, if x=5 and z=64 then y must be

x1 = 5; z1 = 64;

y1 = SolveA$\frac{6\ x1}{29} + \frac{38\ y}{29}$ == z1, yE

{{y -> $\frac{913}{19}$}}

Going back to the original basis

SolveA9a + 4 b == 5, 9 a + 7 b == $\frac{913}{19}$=, 8a, b<E

{{a -> $\frac{103}{19}$, b -> -($\frac{2}{19}$)}}