Lecture II: Applications of Mathematical Programming in Agricultural Economics

I. Interregional and Spatial Economics

A. A large section of the literature, but I only want to discuss two types:

1. Arbitrage or regional supply and demand models. If demand were distributed identically across all areas of the country, but there was a primary supply region (for example the orange juice market). What would the equilibrium look like? What is the impact of profit on a change in transportation costs? What happens if Brazil enters the market?

2. Plant location models. In the past most heavy industry was located in the north, particularly in the area around the great lakes. As the population shifted south more industry shifted south. Why? Current topics, the technology is being developed to condense most of the water out of milk, reducing its shipping cost. Will this cause the dairies to move to Wisconsin? If you were John Deere how do you determine where to locate your warehouses?

B. Leibnitz's Rule

$$V(r)=\int_{A(r)}^{B(r)} f(x,r)\,dx$$

$$V'(r)=\frac{\partial V(r)}{\partial r}=f\left(B(r),r\right)\frac{\partial B(r)}{\partial r}-f\left(A(r),r\right)\frac{\partial A(r)}{\partial r}+\int_{A(r)}^{B(r)}\frac{\partial f(x,r)}{\partial r}dx$$

1. In a market equilibrium

$$\max_x \int_0^x p^d(z)\,dz-\int_0^x p^s(z)\,dz$$

a. $p^d(z)$ is the consumer's inverse demand curve and $p^s(z)$ is the producer's supply curve.

b. This amounts to maximizing the sum of consumer surplus and producer surplus.

a. Differentiating the sum of consumer surplus and producer surplus yields

$$p^d(z)-p^s(z)=0$$

b. Extending the problem

$$\max_{x_T, x_1, x_2} \int_0^{x_1} p_1^d(z)\,dz + \int_0^{x_2} p_2^d(z)\,dz - \int_0^{x_T} p^s(z)\,dz - tx_1$$
$$s.t. \quad x_1 + x_2 \leq x_T$$
$$\Rightarrow \frac{\partial S}{\partial x_1} = p_1^d(x_1) - p^s(x_1 + x_2) = 0$$
$$\frac{\partial S}{\partial x_2} = p_2^d(x_2) - p^s(x_1 + x_2) - t = 0$$

II. **Econometrics and Statistical Applications**

A. Historically, econometrics relied on closed form solutions made possible by linear models of normally distributed random variables.

B. More complicated models that introduce factors such as concavity constraints and nonnormality do not imply closed form solutions.

C. Concavity constrained cost functions:

1. Basic cost function formulation:

$$\left.\begin{aligned} &\min_x w'x \\ &s.t. F(x,y) = 0 \end{aligned}\right\} \Rightarrow C(w,y)$$
$$C(w,y) = \alpha_0 + \alpha' w + \tfrac{1}{2} w' \mathrm{A} w + \beta' y + \tfrac{1}{2} y' \mathrm{B} y + w' \Gamma y$$

2. In this formulation, both the A and B matrices are symmetric by Young's theorem. In addition, if the optimization conditions are met (as we will describe in this course) the cost function is concave in input prices, implying that the A matrix is negative semi-definite.

3. Fitting the cost function:

a. We estimated this cost function using concentrated maximum likelihood.

b. Specifically, the residual vector based on any set of parameter estimates is

$$e_i(\theta) = \begin{bmatrix} c_i - \left(\alpha_0 + \alpha' w_i + \tfrac{1}{2} w_i' \mathrm{A} w_i + \beta' y_i + \tfrac{1}{2} y_i' \mathrm{B} y_i + w_i' \Gamma y_i\right) \\ x_{1i} - \left(\alpha_1 + \mathrm{A}_{11} w_{1i} + \mathrm{A}_{12} w_{2i} + \mathrm{A}_{13} w_{3i} + \Gamma_{11} y_{1i} + \Gamma_{21} y_{2i} + \Gamma_{31} y_{31}\right) \\ x_{2i} - \left(\alpha_2 + \mathrm{A}_{21} w_{1i} + \mathrm{A}_{22} w_{2i} + \mathrm{A}_{23} w_{3i} + \Gamma_{12} y_{1i} + \Gamma_{22} y_{2i} + \Gamma_{32} y_{31}\right) \\ x_{3i} - \left(\alpha_3 + \mathrm{A}_{31} w_{1i} + \mathrm{A}_{32} w_{2i} + \mathrm{A}_{33} w_{3i} + \Gamma_{13} y_{1i} + \Gamma_{23} y_{2i} + \Gamma_{33} y_{31}\right) \end{bmatrix}$$

c. These parameters are estimated by maximizing

$$\max_\theta -\frac{T}{2} \ln\left|\Omega(\theta)\right|$$
$$s.t. \Omega(\theta) = \frac{1}{T} \sum_{i=1}^{T} e_i(\theta)' e_i(\theta)$$

d. While this formulation is complex, it can be estimated using iterative generalized least squares without resorting to complex mathematical programming algorithms.

e. However, one approach to estimating a concavity constrained const function is to estimated the Cholesky decomposition of the A matrix. Specifically,

$$A = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{12} & \alpha_{22} & 0 \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ 0 & \alpha_{22} & \alpha_{23} \\ 0 & 0 & \alpha_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_{11}\alpha_{11} & \alpha_{11}\alpha_{12} & \alpha_{11}\alpha_{13} \\ \alpha_{11}\alpha_{12} & \alpha_{12}\alpha_{12} + \alpha_{22}\alpha_{22} & \alpha_{12}\alpha_{13} + \alpha_{22}\alpha_{23} \\ \alpha_{11}\alpha_{13} & \alpha_{12}\alpha_{13} + \alpha_{22}\alpha_{23} & \alpha_{13}\alpha_{13} + \alpha_{23}\alpha_{23} + \alpha_{33}\alpha_{33} \end{bmatrix}$$

f. This specification is nonlinear enough to require the use of mathematical programming algorithms.

III. Policy Analysis

A. Depending on the scope of the policy analysis, mathematical programming analysis may imply either partial or general equilibrium formulations.

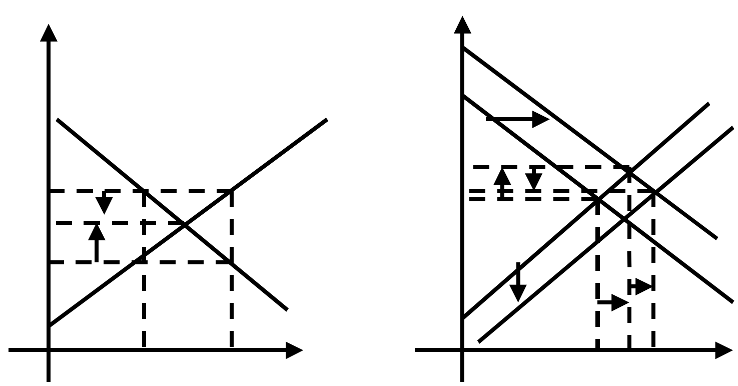
1. Partial equilibrium analysis analyzes the impact of policy changes (such as changes to price floors or export enhancement) on the market-clearing price within a single market.

a. Recent studies have analyzed the impact of additional identity preservation costs imposed on marketing channels by the release of genetically modified organisms.

2. General equilibrium analysis examines the impact of policy changes on the system of prices in the economy.

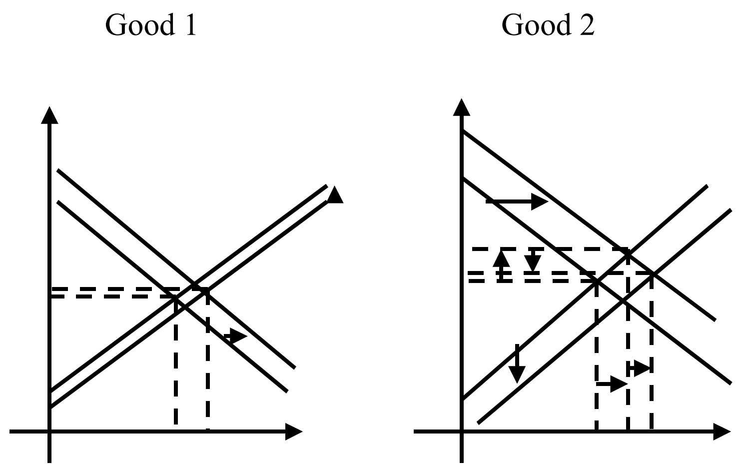
a. The only problem is that we know that other prices change as we remove the price floor.

Good 1 Good 2



The demand for good 2 shifts outward if good 1 and good 2 are substitutes (for the consumer the effect of the removal of the price floor is a price increase.) On the supply side, the decreased price of good 1 leads to an outward shift in supply for good 2 given that the production of good 1 and good 2 compete for limited resources.

b. This interaction is complicated as the first market reacts. Specifically, the increased price of good 2 leads to a leftward shift in the supply curve in the market for good 1 and an upward shift in the demand for good 1.



c. These interactions are further complicated by changes in the factor market. Specifically, if the industry represented a significant demand for the variable factor, the initial elimination of the price floor implies a decline in the value of the input. Hence, the overall demand for the input shifts to the left.

3. Modeling the interaction between the two markets involves moving from a partial equilibrium analysis to a general equilibrium analysis.

a. Early work on general equilibrium analysis involves the concept of a Walrasian equilibrium. The primary idea of the Walrasian equilibrium was the concept that some price vector could be found for any endowment that equated the supply and demand, or resulted in zero excess demand:

$$\xi_i(p) = D(p,W) - S(p,w)$$
$$p_i(\xi_i(p) - W_i) = 0$$
$$\xi_i(p) - W_i \leq 0$$

b. $\xi_I(p)$ is the excess demand for good i, it is a function of the price vector. Demand is determined by the initial endowment of goods W.

b. $p_i(\xi_i(p)-W_i)$ is the complementary slackness conditions. This condition implies that either the price of the i^{th} good is zero, or its excess demand is zero.

IV. Nonparametric Efficiency Analysis

A. A final agricultural specification that has been growing in popularity is the nonparametric analysis of production efficiency.

1. Economic efficiency of farms and agribusinesses has been analyzed by first estimating a parametric cost or profit function as developed in Featherstone and Moss.
2. However, the efficiency results in these studies are conditioned on the choice of structural form used to estimate the parametric production function as well as the distributional assumptions used in estimation.
3. An alternative approach is to allow the most efficient firms to form an efficient technological envelope. Assuming that a firm produces a vector of m output from n inputs, a vector of outputs, y^*, could be produced using some combination of outputs from firms in the sample.
4. Mathematical Specification

$$\min_z \quad c^* = \sum_{i=1}^{n} c_i z_i$$

$$st \quad \sum_{i=1}^{n} y_{ij} z_i \geq y_j^* \;\; j = 1, \cdots m$$

$$\sum_{i=1}^{n} z_i = 1$$