I. Basic Microeconomic Problem

A. The Microeconomic Problem

1. The typical definition of economics is the study of the allocation of scarce resources to satisfy unlimited and competing human wants and desires. Within this basic definition, we see the elements of a mathematical programming model:
    a. An objective function, or a measure of human wants and desires which we will attempt to maximize.
    b. Constraints or limits to simply giving humans all that they could possibly want.
    c. Implied in the definition is also the concept that we have choice variables or instruments within our control to effect this maxmization.
2. Within the consumer's problem we define those elements:

$$\text{Max } U(x)$$
$$s.t. \, x'p \le Y$$

    a. In this problem, the objective function is the consumer's utility, the constraint is income, and the choice variables are levels of different commodities consumed.
    b. How do we determine the optimum for this problem?
3. Similarly, the producer's problem becomes

$$\text{Max } y'p - w'x$$
$$s.t. \; y = F(x)$$

    a. As in the consumer's problem, we have an objective function which is profit, constraints which is the production function, and choice variables which are the levels of inputs and outputs.
    b. Again, how do we determine the optimum in theory and in practice.

Moss, Charles B. "Applied Optimization in Agriculture." In *Handbook of Applied Optimization* edited by Panos Pardolous and Mauricio G. C. Resende, pp. 957–66. New York: Oxford Press, 2002.

II. Agricultural Optimization Problems

A. Food and Diet Problems

1. The food and diet research can be characterized by two major focuses:
    a. Least cost combination of foods to meet dietary needs. Stigler's "Cost of subsistence".
    b. Least cost feed ration studies.
2. The basic application would involve minimizing the cost of a diet subject to some nutritional constraint:

$$\min_x \; c'x$$
$$s.t. \; Ax \geq b$$

a. c is a vector of prices for each food,
b. x is a vector of choice levels for each food,
c. A is a matrix of nutrients provided by each food, and
d. b is a vector of minimum nutritional requirements.

2. More advanced formulations of the diet problem have been developed in the guise of the household production model.
    a. General form household production problem:
$$\max_{y,x} U(y)$$
$$s.t. \; y \leq F(x)$$
$$p'x \leq I$$
        (i) where $F(x)$ denotes the production relationship between purchased foodstuffs and consumable goods (y).
        (ii) p is the price vector for purchased foodstuffs, and
        (iii) I is income.
    b. A linear formulation of such a model can be expressed as
$$\max_{y,x} U(y)$$
$$s.t. \; y \leq Ax$$
$$p'x \leq I$$
    c. In addition to foodstuffs, x can be augmented to include labor use.

B. Farm and Agribusiness Management
1. Initially, linear programming was used to find optimal crop mix. This work has grown into large extension farm planning efforts such as OK farms. These models tend to be either general linear or integer.
$$\max_x c'x$$
$$s.t. \; Ax \leq b$$
    a. x could be a vector of possible crop alternatives (wheat, cotton, and oats),
    b. c was a conformable vector of net returns from each crop activity,
    c. A is a matrix of resource constraints
$$A = \begin{bmatrix} 1 & 1 & 1 \\ .2 & .3 & .1 \\ 25 & 100 & 10 \end{bmatrix} \rightarrow \begin{matrix} Land \\ Labor \\ Capital \end{matrix}$$
    d. b is the vector of resource constraints.
2. This basic formulation has been complicated in several ways.

a. In the Midwest, crop yields are dependent on the length of the growing season. Further, this growing season is determined by the number of field days. However, the number of field days is stochastic because they depend on the weather. One way frequently used to control the effect of the weather is to increase the size of the equipment. Extending the machinery section and integrating a machinery and labor compliment lead to B-10 in Indiana that is a machinery-planning model.

b. Another extension of the mathematical programming approach has been the incorporation of risk into the farm-planning model.

(i) One way that risk may enter the farm management model is by complicating the objective function:

$$\max_x E\left[U(x)\right]$$
$$s.t. \quad Ax \leq b$$

where $E[.]$ is the expected value operator, $U(.)$ is the utility function, A is the resource coefficients, b is the vector of resource constraints, and x is the level of each activity.

(ii) Freund shows that given that preferences are negative exponential and returns are normally distributed, the expected utility function becomes:

$$\left.\begin{array}{r} U(x) = -\exp(-\rho x) \\ x \sim N(\mu, \Omega) \end{array}\right\} \Rightarrow E\left[U(x)\right] = \mu' x - \frac{\rho}{2} x' \Omega x$$

Therefore, the maximization problem becomes a nonlinear optimization problem

$$\max_x \; \mu' x - \frac{\rho}{2} x' \Omega x$$
$$s.t. \quad Ax \leq b$$

(iii) However, given that few closed form conjugates exist, technologies have evolved to allow direct optimization of more generalized problems:

$$U(y) = \frac{y^\lambda}{\lambda}$$
$$y \sim N(\mu' x, x' \Omega x)$$

- Numerical quadrature
- Gaussian quadrature

3. Agribusiness firms benefit from optimization techniques in several ways:
   a. Inventory Management
   b. Transhipment Models
   c. Least Cost Feeds

4. Other applications which emphasize the dynamic nature of decision making include machinery replacement models which are based on Bellman's principle of optimality.

C. Farm Firm Development

1. The typical farm firm development model is primarily interest in firm growth.
   a. Multiperiod programming allowed researchers to examine questions related to investment, capital accumulation, and debt choice.
      (i) Swanson, and Loftsgard and Heady focused on the managerial or production response problem.
      (ii) Later Irwin and Baker, and Martin and Plaxico focused on the financial aspects of growth.
      (iii) Moss (1984) focused on the effect of tax codes on firm growth while Moss (1987) focused on the effect of exogenous factors such as macroeconomic innovations on firm growth.
      (iv) Early studies in this area were deterministic, obviously excluding an important aspect of firm growth. Johnson, Tefertiller, and Moore opened the door by allowing crop yields to be stochastic. Moss (1987) drew from other techniques to approximate stochastic optimal control.
   b. A significant problem with these studies have typically been the size of the matrix.
2. Mathematical specification

$$\max \begin{bmatrix} c_1 & c_2 & c_3 & \cdots & c_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

$$s.t. \begin{bmatrix} A_{11} & 0 & 0 & \cdots & 0 \\ T_{12} & A_{22} & 0 & \cdots & 0 \\ 0 & T_{23} & A_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$$

Focusing on the first two constraints

$$A_{11} x_1 \leq b_1$$

$$T_{12} x_1 + A_{22} x_2 \leq b_2 \Rightarrow A_{22} x_2 \leq b_2 - T_{12} x_1$$

So decisions made in year 1 could also affect the resources available in year 2.

3. Again generalizing the model, a decision in year 1 may have multiple possible outcomes:

$$\max \begin{bmatrix} c_{11} & p_1 c_{12} & p_2 c_{22} & p_3 c_{32} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \\ x_{32} \end{bmatrix}$$

$$s.t. \begin{bmatrix} A_{11} & 0 & 0 & 0 \\ T_{12} & A_{12} & 0 & 0 \\ T_{22} & 0 & A_{22} & 0 \\ T_{32} & 0 & 0 & A_{32} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

a. p_1 denotes the probability of event 1 occurring, p_2 is the probability of event 2 occurring, and p_3 is the probability of event 3 occurring.

b. If event 1 occurs, the profit vector c_{12} and T_{12} resources transfer to period 2.

$$A_{12} x_{12} \leq b_{12} + T_{12} x_{11}$$
$$A_{22} x_{22} \leq b_{22} + T_{22} x_{11}$$
$$A_{32} x_{32} \leq b_{32} + T_{32} x_{11}$$

D. Production Response

1. Production response models have been used to study the impact of some policy or external shock to the sector.

a. From the firm level, the effect of changing fertilizer prices, labor availability, or support prices on firm outputs, profits, and input demands can be mapped out, much like the duality approach to production.

b. The firm level effects are then aggregated to the sector level.

c. An example of this type of model is CARD.

E. Interregional and Spatial Economics

1. A large section of the literature, but I only want to discuss two types:

a. Arbitrage or regional supply and demand models. If demand were distributed identically across all areas of the country, but there was a primary supply region (for example the orange juice market). What would the equilibrium look like? What is the impact of profit on a change in transportation costs? What happens if Brazil enters the market?

b. Plant location models. In the past most heavy industry was located in the north, particularly in the area around the great lakes. As the population shifted south more industry shifted south. Why? Current topics, the technology is being developed to condense most of the water out of milk, reducing its shipping cost. Will this cause the dairies to move to Wisconsin? If you were John Deere how do you determine where to locate your warehouses?

2. Leibnitz's Rule

$$V(r) = \int_{A(r)}^{B(r)} f(x,r)\,dx$$

$$V'(r) = \frac{\partial V(r)}{\partial r} = f\left(B(r),r\right)\frac{\partial B(r)}{\partial r} - f\left(A(r),r\right)\frac{\partial A(r)}{\partial r} + \int_{A(r)}^{B(r)} \frac{\partial f(x,r)}{\partial r}\,dx$$

3. In a market equilibrium

$$\max_x \int_0^x p^d(z)\,dz - \int_0^x p^s(z)\,dz$$

a. $p^d(z)$ is the consumer's inverse demand curve and $p^s(z)$ is the producer's supply curve.

b. This amounts to maximizing the sum of consumer surplus and producer surplus.

c. Differentiating the sum of consumer surplus and producer surplus yields

$$p^d(z) - p^s(z) = 0$$

d. Extending the problem

$$\max_{x_T, x_1, x_2} \int_0^{x_1} p_1^d(z)\,dz + \int_0^{x_2} p_2^d(z)\,dz - \int_0^{x_T} p^s(z)\,dz - tx_1$$

$$s.t.\quad x_1 + x_2 \leq x_T$$

$$\Rightarrow \frac{\partial S}{\partial x_1} = p_1^d(x_1) - p^s(x_1 + x_2) = 0$$

$$\frac{\partial S}{\partial x_2} = p_2^d(x_2) - p^s(x_1 + x_2) - t = 0$$