

AEB 6533 Assignment 3

- I. Assuming a negative exponential utility function and a normal distribution, yields a Expected Value-Variance formulation of the expected utility problem. The objective function for this formulation is

$$\begin{aligned} \max_x \quad & \mu'x - \frac{\rho}{2} x'\Omega x \\ \text{s.t.} \quad & Ax \leq b \end{aligned}$$

where μ is the expected return per acre, x is the choice vector containing the acres planted in each crop, ρ is the Arrow-Pratt risk aversion coefficient, Ω is the variance matrix for returns per acre, and A is the constraint matrix. Assuming a risk aversion coefficient of 0.00002, 640 acres of cropland, \$44,000 of capital, and 100 hours of labor, is the crop combination 436.92 acres of corn, 55.38 acres of peanuts, and 147.69 acres of soybeans optimum? Remember each crop variable must be greater than or equal to zero.

Table 1. Farm Portfolio Problem

	Cotton	Corn	Wheat	Peanuts	Soybeans
Yield	1500	150	45	1.25	45
Price	0.49	2.23	3.46	678.00	6.20
Variable Cost	450	150	50	500	75
Capital	225.00	50.00	33.33	333.33	25.00
Labor	1.00	0.10	0.05	0.75	0.10
Expected Profit per Acre	285.00	184.50	105.70	347.50	204.00
<i>Variance</i>					
Cotton	5795.2	2814.8	1655.8	4967.3	5298.4
Corn	2814.8	1367.2	804.2	2412.7	2573.5
Wheat	1655.8	804.2	473.1	1419.2	1513.8
Peanuts	4967.3	2412.7	1419.2	4257.7	4541.5
Soybeans	5298.4	2573.5	1513.8	4541.5	4844.3

- II. Assume that the cost function for a household production model can be written as

$$c(m) = 440.74 + (0.003 \quad 0.016 \quad 0.009) \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} +$$

$$(m_1 \quad m_2 \quad m_3) \begin{pmatrix} 0.050 & 0.000 & -0.003 \\ 0.000 & 0.060 & -0.002 \\ -0.003 & -0.002 & 0.075 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

Assuming that the consumer's utility function can be written as

$$U(m) = m_1^3 m_2^4 m_3^3$$

and the consumer's cost function must be less than \$450.00, is the vector $m_1 = 7.589$, $m_2 = 7.852$, and $m_3 = 6.282$ optimum?