AEB 6533
Assignment 2

Assuming a set of demand elasticities (ε^1), a set of supply elasticities (η^1), equlibrium prices (p_1), and equlibrium quantities (q_1) in country 1:

$$\varepsilon^1 = \begin{pmatrix} -1.50 & 0.75 \\ 0.61 & -0.50 \end{pmatrix} \qquad p_1 = \begin{pmatrix} 3.50 \\ 5.70 \end{pmatrix}$$

$$\eta^1 = \begin{pmatrix} 0.50 & -0.05 \\ -0.04 & 0.30 \end{pmatrix} \qquad q_1 = \begin{pmatrix} 1000 \\ 750 \end{pmatrix}$$

along with a set of demand elasticities (ε^2), a set of supply elasticities (η^2), equlibrium prices (p_2), and equlibrium quantities (q_2) in country 1:

$$\varepsilon^2 = \begin{pmatrix} -1.25 & 0.80 \\ 1.08 & -0.75 \end{pmatrix} \qquad p_2 = \begin{pmatrix} 3.25 \\ 6.00 \end{pmatrix}$$

$$\eta^2 = \begin{pmatrix} 0.60 & -0.07 \\ -0.09 & 0.40 \end{pmatrix} \qquad q_2 = \begin{pmatrix} 1250 \\ 500 \end{pmatrix}$$

the social welfare can be expressed as:

$$\max_{\substack{q_{11}^s, q_{11}^d, q_{12}^s, q_{12}^d, \\ q_{21}^s, q_{21}^d, q_{22}^s, q_{22}^d}} \left[\left(\tilde{a}_1 q_1^d + \tfrac{1}{2} q_1^{d\prime} \tilde{A}_1 q_1^d \right) - \left(\tilde{b}_1 q_1^s + \tfrac{1}{2} q_1^{s\prime} \tilde{B}_1 q_1^s \right) \right] +$$

$$\left[\left(\tilde{a}_2 q_2^d + \tfrac{1}{2} q_2^{d\prime} \tilde{A}_2 q_2^d \right) - \left(\tilde{b}_2 q_2^s + \tfrac{1}{2} q_2^{s\prime} \tilde{B}_2 q_2^s \right) \right]$$

$$q_1^d = \begin{pmatrix} q_{11}^d \\ q_{12}^d \end{pmatrix}, q_1^s = \begin{pmatrix} q_{11}^s \\ q_{12}^s \end{pmatrix}, q_2^d = \begin{pmatrix} q_{21}^d \\ q_{22}^d \end{pmatrix}, q_2^s = \begin{pmatrix} q_{21}^s \\ q_{22}^s \end{pmatrix}$$

$$s.t.\ q_{11}^d - q_{11}^s + q_{21}^d - q_{21}^s = 0$$

$$q_{12}^d - q_{12}^s + q_{22}^d - q_{22}^s = 0$$

where

$$\tilde{a}_1 = \begin{pmatrix} 14.252 & 33.119 \end{pmatrix} \quad \tilde{A}_1 = \begin{pmatrix} -0.00478 & -0.00796 \\ -0.00796 & -0.0259 \end{pmatrix}$$

$$\tilde{b}_1 = \begin{pmatrix} -4.780 & 15.140 \end{pmatrix} \quad \tilde{B}_1 = \begin{pmatrix} 0.007097 & 0.001577 \\ 0.001577 & 0.025684 \end{pmatrix}$$

$$\tilde{a}_2 = \begin{pmatrix} 74.372 & 203.658 \end{pmatrix} \quad \tilde{A}_2 = \begin{pmatrix} -0.02753 & -0.07342 \\ -0.07342 & -0.21178 \end{pmatrix}.$$

$$\tilde{b}_2 = \begin{pmatrix} -3.296 & -11.864 \end{pmatrix} \quad \tilde{B}_2 = \begin{pmatrix} 0.004457 & 0.00195 \\ 0.00195 & 0.030853 \end{pmatrix}$$

1. Compute the gradient of the objective function at the original solution.
2. What is the nullspace for the constraints?
3. Compute the projected gradient at the original solution.
4. Is the current solution optimal?
5. Are the second-order necessary conditions met?