Farm Portfolio Problem: Part II
Lecture VI

I. Hazell, P.B.R. "A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning Under Uncertainty." *American Journal of Agricultural Economics* 53(1971):53-62.

   A. This article is the basis for the application of MOTAD (Minimize Total Absolute Deviation) in agriculture.

      1. Hazell's approach is two fold. He first sets out to develop review expected value/variance as a good methodology under certain assumptions.

      2. Then he raises two difficulties.

         a. The first difficulty is the availability of code to solve the quadratic programming problem implied by EV.

         b. The second problem is the estimation problem. Specifically, the data required for EV are the mean and the variance matrix. However, the variance matrix is an artifact of the assumption of normality.

   B. The crux of the estimation problem is that the covariance terms in the EV formulation are estimated by:

$$\sum_{j=1}^{n}\sum_{k=1}^{n} x_j x_k \left[\frac{1}{s-1}\sum_{h=1}^{s}(c_{hj}-g_j)(c_{hk}-g_k)\right]$$

   where x_j is the level of activity j in the portfolio, c_{hj} is the observed return on asset j at time h, g_j is the expected return on asset j, s is the number of observations, and n is the number of assets. This equality can be reformulated as:

$$s^2 = \frac{1}{s-1}\sum_{h=1}^{s}\left[\sum_{j=1}^{n} c_{hj} x_j - \sum_{j=1}^{n} g_j x_j\right]^2 .$$

   Hazell suggests replacing this objective function with the mean absolute deviation

$$A = \frac{1}{s}\sum_{h=1}^{s}\left|\sum_{j=1}^{n}\left(c_{hj}-g_j\right)x_j\right| .$$

   Thus, instead of minimizing the variance of the farm plan subject to an income constraint, you can minimize the absolute deviation subject to an income constraint. Another formulation for this objective function is to let each observation h be represented by a single row

$$y_h = \sum_{j=1}^{n}\left(c_{hj}-g_j\right)x_j$$

$$y_h^+ - y_h^- = \sum_{j=1}^{n}\left(c_{hj}-g_j\right)x_j$$

   where y_h is the deviation from average. This deviation can be divided into positive deviations from the average, y_h^+, and negative deviations from the average, y_h^-. The complete mathematical programming problem can then be written as:

$$\min_x sA = \sum_{h=1}^{s} \left(y_h^+ + y_h^- \right)$$

$$st \sum_{j=1}^{n} \left(c_{hj} - g_j \right) x_j - y_h^+ + y_h^- = 0$$

$$\sum_{j=1}^{n} g_j x_j = l$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$

A last modification comes from the refocusing on the negative deviations. Thus, the problem becomes:

$$\min_x sA = \sum_{h=1}^{s} y_h^-$$

$$st \qquad \sum_{j=1}^{n} \left(c_{hj} - g_j \right) x_j + y_h^- \geq 0$$

$$\sum_{j=1}^{n} g_j x_j = l$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$

C. A Numerical Example:

Hazell's Florida Farm

	Carrots	Celery	Cucumbers	Peppers
1	292	-128	420	579
2	179	560	187	639
3	114	648	366	379
4	247	544	249	924
5	426	182	322	5
6	259	850	159	569
Average	253	443	284	516
1	39	-571	136	63
2	-74	117	-97	123
3	-139	205	82	-137
4	-6	101	-35	408
5	173	-261	38	-511
6	6	407	-125	53

The Farm Planning Model

$$\min_x \quad y_1^- + y_2^- + y_3^- + y_4^- + y_5^- + y_6^-$$

$$\begin{array}{lcr}
x_1 + x_2 + x_3 + x_4 & \leq & 200 \\
25x_1 + 36x_2 + 27x_3 + 87x_4 & \leq & 10000 \\
-x_1 + x_2 - x_3 + x_4 & \leq & 0 \\
39x_1 - 571x_2 + 136x_3 + 63x_4 + y_1^- & \geq & 0 \\
-74x_1 + 117x_2 - 97x_3 + 123x_4 + y_2^- & \geq & 0 \\
-139x_1 + 205x_2 + 82x_3 - 137x_4 + y_3^- & \geq & 0 \\
-6x_1 + 101x_2 - 35x_3 + 408x_4 + y_4^- & \geq & 0 \\
173x_1 - 261x_2 + 38x_3 - 511x_4 + y_5^- & \geq & 0 \\
6x_1 + 407x_2 - 125x_3 + 53x_4 + y_6^- & \geq & 0 \\
253x_1 + 443x_2 + 284x_3 + 516x_4 & = & l
\end{array}$$

Specs File

```
BEGIN HAZELL
     MINIMIZE
     LIST                          200
     OBJECTIVE                     A
     RHS                           RHS01
*     BOUNDS                        BOUND01
     VERIFY GRADIENTS
     SOLUTION FILE                 20
END HAZELL
NAME           HAZELL
ROWS
 N  A
 L  LAND
 L  LABOR
 L  ROTATE
 G  OUTCOM1
 G  OUTCOM2
 G  OUTCOM3
 G  OUTCOM4
 G  OUTCOM5
 G  OUTCOM6
 E  INCOME
COLUMNS
    X1        LAND          1.0         LABOR            25.0
    X1        ROTATE       -1.0         OUTCOM1          39.0
    X1        OUTCOM2     -74.0         OUTCOM3        -139.0
    X1        OUTCOM4      -6.0         OUTCOM5         173.0
    X1        OUTCOM6       6.0         INCOME          253.0
    X2        LAND          1.0         LABOR            36.0
    X2        ROTATE        1.0         OUTCOM1        -571.0
    X2        OUTCOM2     117.0         OUTCOM3         205.0
    X2        OUTCOM4     101.0         OUTCOM5        -261.0
    X2        OUTCOM6     407.0         INCOME          443.0
    X3        LAND          1.0         LABOR            27.0
    X3        ROTATE       -1.0         OUTCOM1         136.0
```

```
    X3        OUTCOM2    -97.0          OUTCOM3          82.0
    X3        OUTCOM4    -35.0          OUTCOM5          38.0
    X3        OUTCOM6   -125.0          INCOME          284.0
    X4        LAND         1.0          LABOR            87.0
    X4        ROTATE       1.0          OUTCOM1          63.0
    X4        OUTCOM2    123.0          OUTCOM3        -137.0
    X4        OUTCOM4    408.0          OUTCOM5        -511.0
    X4        OUTCOM6     53.0          INCOME          516.0
    Y1        A            1.0          OUTCOM1           1.0
    Y2        A            1.0          OUTCOM2           1.0
    Y3        A            1.0          OUTCOM3           1.0
    Y4        A            1.0          OUTCOM4           1.0
    Y5        A            1.0          OUTCOM5           1.0
    Y6        A            1.0          OUTCOM6           1.0
RHS
    RHS01     LAND       200.0          LABOR         10000.0
    RHS01     INCOME   62729.0
ENDRUN
```

II. Focus-Loss

A. Two factors make Focus-Loss acceptable

1. First, like Hazell's MOTAD, the Focus-Loss problem is solvable using linear programming.
2. Second, Focus-Loss has a direct appeal in that it focuses attention on survivability

B. The first step in the Focus-Loss methodology is to define the maximum allowable loss

$$L = E(z) - z_c = \sum_{j=1}^{n} E(c_j)x_j - E(F) - z_c$$

a. L - Maximum allowable loss
b. E(z) - Expected income for the firm
c. z_c - Required cash income
d. $E(c_j)$ - Expected income from each crop, j
e. x_j - Level of the j*th* crop (activity)
f. E(F) - Expected level of fixed cost

C. Given this definition, the next step is to define the maximum deficiencies or loss arising from activity j.

$$r_j = E(c_j) - r_j^*$$

where r_j^* is the worst expected outcome. For example, a crop failure may give an r_j of -$100 which would represent your planting cost. Given this potential loss, the Focus-Loss scenario is based on restricting the largest expected loss to be above some stated level

$$r_r x_j \le \frac{L}{k}$$

D. In the Anderson, Dillon and Hardaker example:

$$\max_x \; 72\,x_1 + 53.4\,x_2 + 88.8\,x_3 - 200$$

$$\begin{array}{lllll}
x_1 + & x_2 + & x_3 & & \le 12 \\
x_1 & + & x_3 & & \le 8 \\
30\,x_1 + & 20\,x_2 + & 40\,x_3 & & \le 400 \\
5\,x_1 + & 5\,x_2 + & 8\,x_3 & & \le 80 \\
60\,x_1 & & & -\frac{1}{3}L & \le 0 \\
 & 44.5\,x_2 & & -\frac{1}{3}L & \le 0 \\
 & & 74\,x_3 & -\frac{1}{3}L & \le 0
\end{array}$$

1. The choice of k=3 is somewhat arbitrary.
   a. Two points about the Focus-Loss
      i. Allowing L -> -Infinity is the profit maximizing solution
      ii. L can become large enough to make the linear programming problem infeasible.
   b. A Better Justification for k
      i. One alternative for setting k results from the notion that

$$r_j^* = m_j - t_p\, s_j$$

      ii. Thus, if we let t_p be -1.96, the maximum loss would be 1.96 σ_j

$$(s_j t_p)\, x_j \le \frac{L}{k}$$

II. Direct Utility Maximization

A. To this point, we have discussed several alternatives to direct utility maximization which were based on efficiency criteria (as in the case of expected value-variance) or some ad hoc specification of risk aversion as in the case of focus-loss.

B. Next, I want to discuss the application of a discrete form of expected utility analysis

	Corn	Soybeans	Wheat
Observation 1	176.24	94.81	97.09
Observation 2	232.93	114.39	120.18
Observation 3	273.01	144.50	108.75
Observation 4	221.59	114.32	87.48
Observation 5	-7.87	97.22	100.46
Observation 6	247.59	126.41	108.34
Observation 7	226.79	113.49	98.16
Observation 8	250.11	123.27	107.60
Observation 9	255.99	136.15	102.81
Observation 10	246.91	131.04	104.68
Average	212.33	119.561	103.5551

C. Parameterization of the Direct Expected Utility Model

1. First, we start by maximizing the expected profit given that the total acres do not exceed 1280. This yields an annual profit of \$271,782. Amortizing this amount into perpetuity using a discount rate of 15% yields a total value of \$1,811,880.
2. Assuming the debt-to-asset position of the farm is 60%, the value of the asset represents equity of \$724,752 and debt of \$1,087,130.
3. Assuming an interest rate of 12.5% yields an annual cash flow requirement of \$135,891 to cover the interest payments.
4. Further assuming a family living requirement of \$50,000 yields a minimum cash requirement of \$185,891. Subtracting this from the equity yields a wealth constraint of

$$W_i = 176.24\,x_1 + 94.81\,x_2 + 97.09\,x_3 + 538861$$

5. A simple model of crop choice can then be defined as

$$\max_x \quad \frac{1}{10}\frac{W_1^b}{b} + \frac{1}{10}\frac{W_2^b}{b} + \cdots \frac{1}{10}\frac{W_{10}^b}{b}$$

$$\begin{aligned}
x_1 + \quad x_2 + \quad x_3 & \qquad\qquad\qquad \le 1280 \\
-176.2\,x_1 - 93.8\,x_2 - 97.1\,x_3 + W_1 & = 538{,}861 \\
-232.9\,x_1 - 114.4\,x_2 - 120.2\,x_3 + W_2 & = 538{,}861 \\
& \vdots \\
-246.9\,x_1 - 131.0\,x_2 - 104.7\,x_3 + W_{10} & = 538{,}861
\end{aligned}$$

a. In addition to rearranging the sign on the objective function, we also must scale the problem. Specifically, assuming an average payoff, the terminal wealth will be:

	w	w/1000
u(w,r) = w^r/r with w = 810643, r=-1.0	-1.233E-06	-1.233E-03
u(w,r) = w^r/r with w = 810643, r=-2.0	-7.600E-13	-7.609E-07

$u(w,r) = w^r/r$ with w = 810643, r=-3.0	-6.257E-19	-6.27E-10

In the original column, the objective number is much smaller, which will lead to difficulties in the numerical optimization. As a result, I have substituted wealth defined in thousands of dollars in the second column which helps scale the optimization problem. In addition, I also scale the objective function directly. In doing so, I try to get the optimal value of the objective function to be around -1 to -10.

b. The optimal solutions are then given by the crop rotations

r	x_1	x_2	x_3	μ	σ
-0.001	929.12	350.88	0.00	239,231.70	75,923.56
-0.1	882.59	397.41	0.00	234,915.10	72,865.46
-1.0	532.69	747.31	0.00	202,455.70	50,130.77
-10.0	4.93	0.00	284.59		