Lecture I: Mathematical Economics

I. Mathematical Economics is an approach to economic analysis. The economist draws upon mathematical principles to aid in reasoning.

II. Mathematical versus Non-Mathematical Economics.

A. The major difference between mathematical economics and literary economics is that in mathematical economics assumptions and conclusions are expressed in mathematical terms, while literary economics uses rhetoric (or hermeneutics).

1. Describe in words what a utility function is and how the consumer decides on a consumption bundle.
2. Mathematically,

$$U = U(x_1, x_2)$$
$$Y \geq p_1 x_1 + p_2 x_2$$

which leads to the choice problem:

$$\max_{x_1, x_2} U = U(x_1, x_2)$$
$$s.t. \quad Y \geq p_1 x_1 + p_2 x_2$$

.

In this course, we will demonstrate that maximizing a Lagrangian form of this function can solve this problem:

$$\max_{x_1, x_2} L = U(x_1, x_2) + \lambda(Y - p_1 x_1 - p_2 x_2).$$

Solving this Lagrangian form yields a set of equations relating the endogenous (or choice variable) to exogenous variables:

$$U = U^*(p_1, p_2, Y)$$
$$x_1 = x_1^*(p_1, p_2, Y)$$
$$x_2 = x_2^*(p_1, p_2, Y)$$
$$\lambda = \lambda^*(p_1, p_2, Y)$$

.

However, this solution requires some assumptions. For example the utility function $U(x_1, x_2)$ must be concave with respect to each consumption good.

B. Mathematical economics forces the economist to make assumptions explicit at every stage of reasoning. Mathematical theorems are usually stated in "if-then" form. To make the statement "then", you must satisfy all the "if" conditions.

C. Geometric analysis suffers from serious dimensionality problems.

D. Advantages of the mathematical approach (Chaing p.4).

1. The language is more concise and precise.
2. We have a lot of mathematical tools to draw on.
3. It forces the analyst to explicitly state the assumptions made.
4. It allows for generalization to the *n*-variable case

Example

$C =$ circumference

$C_{BB} = 2\pi R \qquad R = 1\ ft$

$C_{world} = 2\pi R \qquad R = 4000\,mile$

How much circumference to you add to each when you add a foot to the radius?

$$C_{BB+1\,ft} = 2\pi(1\,ft + 1\,ft)$$

$$\Delta C_{BB} = 2\pi(1\,ft + 1\,ft) - 2\pi(1\,ft)$$

$$= 2\pi(1\,ft)$$

$$C_{world+1\,ft} = 2\pi(4000mile + 1\,ft)$$

$$\Delta C_{world} = 2\pi(4000mile + 1\,ft) - 2\pi(4000mile)$$

$$= 2\pi(1\,ft)$$

$$\therefore \Delta C_{BB} = \Delta C_{world} = 2\pi(1\,ft)$$

Intuition was slightly ajar. Mathematics allowed for a concise problem statement and precise solution.

E. Traditional objections to Mathematical Economics
   1. Some have a philosophical objection to the use of a precise tool such as mathematics to an imprecise study like economics.
      a. In the quest to add more precision to the study of economics, wouldn't it make sense to use a precise tool?
      b. At one time chemistry, physics, and astronomy were imprecise studies whose evolution was aided by mathematics.
      c. The argument of imprecise study may mean unknowns or ambiguity not fully investigated that can be aided through recognizing faults.
   2. Some find mathematics difficult and argue that it will never help the understand economics, only confuse.

Pigou "Objections from people innocent of mathematics are like objections to Chinese literature by people who cannot read Chinese."

III. Mathematical Economics versus Econometrics
   A. Econometrics is the empirical study of economic data by use of statistical techniques.
   B. Mathematical economics refers to the application of mathematics to theoretical analysis.

Readings: Chaing Chapter 2:7-31.
Timbrell Chapter 1: 9-27.